Nicole Oresme and the Revival of Medieval Mathematics

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Outline

When teaching the history of mathematics there is a tendency to begin with an introduction of the ancient and hellenistic worlds, then to jump to Enlightenment era Europe and study the works of Galileo, Fermat and Descartes. While there is good reason for teaching the history of math this way it unfortunately overlooks many of the necessary intermediate strides made in the medieval world, which allowed Europe to develop into intellectual maturity. One mathematician whose work goes particularly overlooked due to this bias is French scholastic Nicole Oresme. Nicole Oresme's name is often completely overlooked in the history of mathematics, or only brought up in passing as precursor to more significant mathematicians. But, I will argue that Oresme was a brilliant mind of his own accord, who deserves recognition from the mathematical community as a key figure in the development of Western mathematics. This paper will attempt to demonstrate this by looking at Oresme's contributions to mathematics through his studies of infinite series and his development of early graphing techniques. These works are interesting and significant because there is a clear lineage of intellectual development from Oresme to the great mathematicians of the Renaissance and the Enlightenment, which to overlook would be to give an insufficient accounting of the history of mathematics.

Biography

Some cities in place and time offer a unique mix of social outlook, political motivation, and economic prosperity which, to historians, make them appear destined to create great works of artistic and scientific achievement: 3rd century Alexandria, 16th century Florence, 19th century London, and so on. This absolutely cannot be said of the time and place Nicole Oresme lived and worked. 14th century France was in the middle of the Hundreds Years' War with Britain, they were experiencing the height of the Black Death, which saw as high as 50% mortality in some areas, and the one pillar of stability in Western Europe, the Catholic Church, was experiencing a schism with two popes in Rome and Avignon ([1], 8). From the million-mile view granted to us by history, it is clear there was little to unify the scholars of Europe and motivate great works of scholarly progress. Yet, despite these tumultuous times a few men managed to make tremendous strides in academic progress, and produce great works of scholastic and personal value, which motivated scholars for centuries to come.

As is often the case with medieval archives it is difficult to find much on the early life of Oresme. His name does not appear in any records until 1342, where he is listed as a master of arts at the University of Paris, and there are only two other people with the name Oresme who show up in records from the area he originated from, most likely brothers. Because of this it is assumed he came from a relatively low-class family. Despite the fragmented records of the era, it is generally agreed upon by historians that he was born and raised in Northern France, most likely in the village of Allemagne, and was most likely born around 1323, in order to achieve his status as master of arts by the year he did ([11], 542).

As many scholars of his era were, Oresme was a polymath who studied all of the Carolingian liberal arts. Though there are no records of his schooling, the fact that he would achieve the rank of master and go on to become an accomplished theologian, translator, and mathematician it is safe to assume he excelled at all of his classes. In 1356 he would be promoted to grand master of the College of Navarre at the University of Paris. It is difficult to place exact dates on the publications of his mathematical treatises, but it seems during this time while teaching at Navarre he would publish most of his mathematical work - *Questiones Super*

Geometriam Euclidis, Tractatus de Configuationibus Qualitatum et Motuum, and *Agorismus Proportionum* ([17], 299). While holding the title of grand master he would serve on a number of committees and boards and introduce numerous scholarship programs in attempts to bring more intellect to Northern France, which was difficult due to the war raging on ([1], 10).

He would continue on as grand master until 1362 when he would leave the university and take on an advisor role to the freshly coronated King Charles V of France. Oresme would be a faithful advisor and describe himself as a “humble chaplain ([1], 11).” As reward for his faithful service Charles would reward Oresme handsomely through stipends, gifts, and by increasing the scope of his responsibilities over the years. This is demonstrated when in 1363 Oresme was tasked with heading up a diplomatic envoy to Avignon with the goal of acquiring the support of the pope on behalf of the king ([1], 11).

Oresme’s greatest contributions to his king would without doubt be his intellectual achievements, and most notably his translations of Aristotle and Ptolemy into French. Oresme would translate Aristotle's *Ethics*, *Economics*, *Politics*, and *On the Heavens,* and provide valuable commentary - some of which will be analyzed later in this paper. The king found the insights of Aristotle and Oresme so important he tasked his counselors with reading and studying these works. Oresme also provided his king with a translation of Ptolemy’s *Quadripartitum,* considered the bible of astrology. It appears Charles put a great deal of weight into the idea’s of astrology and mysticism even though Oresme did not. Interestingly enough even though he lived in the middle ages and was surrounded by those who believed deeply in astrology, Oresme himself was highly critical of mysticism and the occult. In 1356 Oresme would publish *Livre de Divinacions*, in which he inveighs astrology, in an attempt to convince the king to put less weight into ideas of the occult, it had no effect ([1], 13). After some years closely serving his King, Charles would help Oresme acquire positions as the archdeacon of the University of Paris and as dean of Rouen Cathedral, very high positions with healthy pay. The final position Oresme would take in his life was the position of Bishop of Lisieux, which he would hold until his death on July 11, 1382 ([1], 15).

Infinite Series

The first contributions of Oresme's to the annals of mathematics this paper will look at is his study of infinite series. There will also be particular emphasis on his proof of the divergence of the harmonic series, which is still taught the same way in introductory calculus textbooks today.

While it would not be completely accurate to claim Ancient Greek mathematicians invented the study of infinite series, it is with the Ancient Greeks where the earliest roots of what would later evolve into the study of infinities can be found. Ancient Greek mathematicians were the first to ask some of the important questions which would set mathematicians down the path of studying infinity and infinite series, but the Greeks themselves were limited by their narrow concepts of numbers and infinity. It is said that the Greeks had *horror infiniti* or a horror of the infinity, and would often attempt to qualify things as greater than forever rather than quantify them into an abstract concept ([4], 241). For example, Euclid would say "Prime numbers are more than any assigned multitude of prime numbers" as opposed to "there exist an infinite number of prime numbers ([6], 271)." Yet, the Greeks saw use in infinite series where they could bound their terms in some geometric conceptual framework.

One of the most notable early problems dealing with infinites and infinite sums from the hellenistic world is Zeno's paradox. Zeno's paradox asks if it is possible for one to travel some distance if they only travel half way at a time. So is it possible to travel from A to B by first traveling half that distance, then half that second distance again, and so on. This can be interpreted in mathematical terms as asking does the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$... converge to 1. The intuitive answer being yes gave enough of a framework to accept that terms approaching infinitely small values can have real world implications. The Greek's interpretation of infinity can be summed up in Aristotle's words when he writes:

> . . . clearly there is a sense in which the infinite exists and another sense in which it does not . . . magnitude is never actually infinite, but it is infinite by way of division—for it is not difficult to refute the theory of indivisible lines—the alternative that remains, therefore, is that the infinite exists potentially. [7]

Eudoxous and Archimedes had also used what modern mathematics would describe as rudimentary infinite series while exploring the method of exhaustion to solve a number of geometric problems ([2], 451) - most notably Archimedes used this method to discover a formula for the area of a parabolic segment by summing the inverse powers of four ([3], 182). But, as with Zeno, all these problems were bounded within some physical, geometric framework. The view proposed by Aristotle dominated the western thought on infinity until medieval scholars diverged from the works of the ancients and proposed new views and problem solving methods. It is also worth noting that similar progress on rudimentary infinite sums had been studied outside of the West in India and China - in the 14th century Indian mathematician Mādhava had discovered he could describe the inverse tangent function as an infinite series, and subsequently used this fact to write π as an infinite series ([3], 183).

Outside of limiting cases where geometric intuition could be used to verify convergence, infinite series did not see much research until interest in them was sparked by intellectuals in Paris and Oxford in the 14th century. The first school to show serious interest in infinite series were the Oxford Calculators at Merton College. These were a group of mathematicians working in 14th century Oxford who were particularly concerned with measurement and motion. One of their most influential figures was Richard Suiseth, whose *Liber Calculationum* (*Book of Calculations*) published circa 1350, would offer the first proof of a series converging which was not geometric in nature ([4], 241). Using a purely verbal argument he was able to show the infinite sum of the natural numbers divided by corresponding powers of two converges to two. This is to say in modern notation:

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \ldots = 2$$

The verbal proof offered by Suiseth while correct is prolix and difficult to follow. Oresme offers a more elegant proof with the aid of figures 1. and 2.

no . after figure nu mumbers

To prove this Oresme begins with a square of length and width one. He then

just say side length 1 since it is a square

divides the area of the larger rectangle in half successive times to create areas of

This is confusing, there is no larger rectangle. Say divide the square in half vertically, then divide one half in half successively as in figure 1a below.

$\frac{1}{2}, \frac{1}{8}, \frac{1}{16}$ and so on. In figure 1. you can see the area of E is $\frac{1}{2}$ the area of F is $\frac{1}{4}$ and the area of G is $\frac{1}{8}$. Because the area of the original shape is one, we know the sum of these terms is one - you may recognize this as another phrasing of Zeno's paradox.

Next, in diagram 1c. he imagines adding each of these areas onto the original shape. This creates a new shape whose area can be expressed by the series

$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \dots \frac{n}{2^n}$, or our desired series of interest. This is perhaps better visualized in figure 2. where it is more clear how each term is being added a successive number of times. Because we know the area of the original square is one, and all of the stacked terms sum to the area of the square, our series sums to twice the area of the original square, or two.

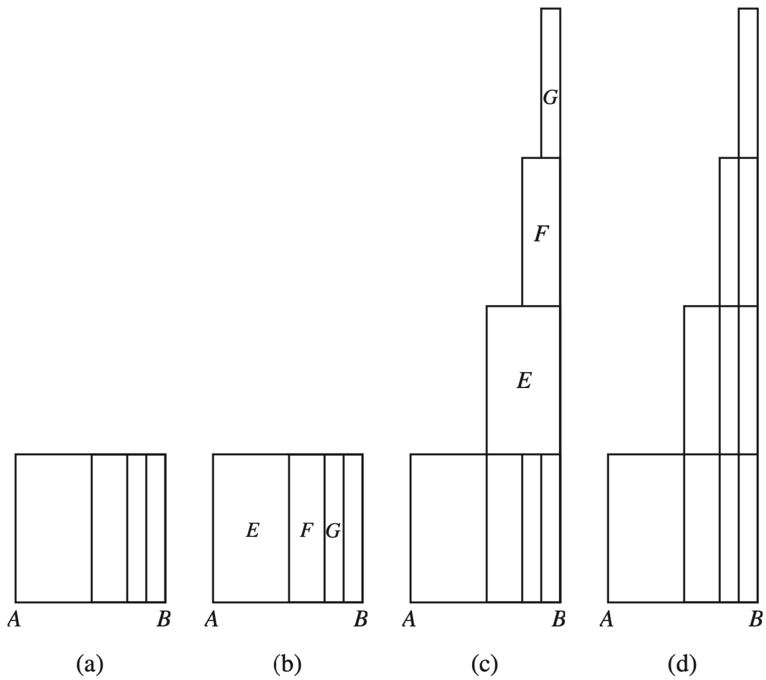

Figure 1. ([12], 31)

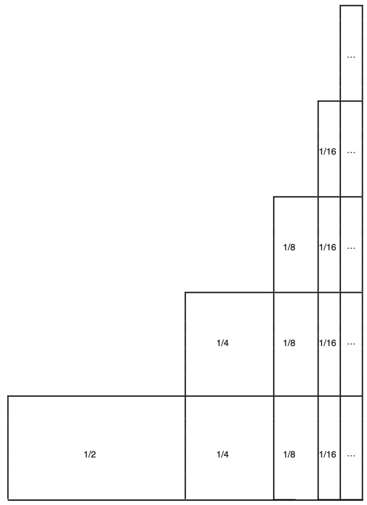

Figure 2. ([4], 454)

Oresme argues that this shape is finite, saying: "A finite surface can be made as long as we wish, or as high, by varying the extension without increasing the size ([3], 183)." Historian John Stillwell notes that this is perhaps the first construction in the history of mathematics of a shape with "infinite extent and finite content ([3], 183)." Oresme used variations on this method to determine the convergence of other series, such as

$\sum_{n=1}^{\infty} \frac{3n}{4^n} = \frac{4}{3}$, and the harmonic series ([4], 241).

The question of the convergence or divergence of the harmonic series asks if the infinite sum of the reciprocals of the natural numbers converges; or in modern notation does this series converge:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \dots$$

The problem seems to have been around in Western thought for some time as it relates questions of geometry, arithmetic, and music theory. The series would not receive the name "harmonic series" until the 18th century when it was noted by Englishman Ephraim

Chambers in his *Chambers Cyclopedia* that the series has elegant harmonic proportions ([5], 203).

While the harmonic series does not provide the same physical constraints as Zeno's walk or Archimedes parabola, using the logic of Aristotle it is easy to see why mathematicians assumed the convergence of the series ([2], 449). However in his third proposition his *Questiones Super Geometriam Euclidis* (*Questions on Euclid's Geometry)* published circa 1350 Oresme gave an elegant proof for the divergence of this series. Oresme states:

> It is possible that an addition could be made, though not proportionally, to any quantity by ratios of lesser inequality, and yet the whole would become infinite. ([5], 202)

Oresme begins with the series written out:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\ldots$$

He then demonstrates that the terms $\frac{1}{3}+\frac{1}{4}>\frac{1}{2}$ this means we can construct a second series where we substitute in this $\frac{1}{2}$ term and it will be less than the original series:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\ldots\geq\frac{1}{1}+\frac{1}{2}+\frac{1}{2}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\ldots$$

Now observe that $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}>\frac{1}{2}$ this means we can construct another series where we substitute in this $\frac{1}{2}$ term and it will be less than the original series:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\ldots\geq\frac{1}{1}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\ldots$$

Oresme demonstrated that you can double the amount of terms collected as much as needed and always have a sum quantity greater than one-half. Because it is obvious that an infinite sum of one-half's diverges and this series is less than our original, our original series must diverge ([2], 449).

This is a particularly remarkable fact when one considers that it would take 10^{43} terms for the series to reach 100 ([15], 104). Unfortunately, following Oresme's death his proof was lost for some centuries and this result would be rediscovered independently by Italian mathematician Pietro Mengoli in 1647 and Swiss mathematician Johann Bernoulli in 1687 using different methods ([5], 203). Johann's brother Jacob was so amazed by his brother's discovery that in 1689 he wrote a poem about the harmonic series:

> So the soul of immensity dwells in minutia. And in narrowest limits no limits adhere. What joy to discern the minute in infinity! The vast to perceive in the small, what divinity! [15, 104]

In the time of Oresme there was not much use for these kinds of problems and they were seen as more of a novelty. However, this work would spark deeper investigations into different types of infinities and motivate mathematicians to develop more rigorous rules for testing convergence, which would be instrumental in the birth of calculus. Solving questions of this manner would become of great interest to mathematicians in coming centuries with the most famous being the Basel problem which asks about the convergence of the reciprocal of the squared numbers.

Graphing and Motion

This section will delve into Oresme's development's of graphs, coordinate systems, and his theories on motion. It is worth noting Oresme's mathematical theories were developed in conjunction with his theological work on ontological forms, which were significant breakthroughs in anti-Aristotelian thought. Unfortunately much of his theological contributions are beyond the scope of this paper, so we will be solely focused on mathematical interpretations of his work.

Oresme did not invent the coordinate system. Coordinate systems had been used since the times of Hipparchus and Strabo to represent astronomical and geographical positions. And, the geometers of ancient times had constructed abstract 2-dimensional planes for the purposes of understanding relationships between geometric shapes. In fact, Ancient Greek geometers such as Menaechmus, and Apollonius had both found that they could define geometric curves in terms of equations. Most notably Menaechmus did this when he formalized the conic sections ([3], 110). Oresme's significance in this field comes from his revelation that he could graph equations, which represented real world phenomena, on 2 perpendicular axis, to construct a geometric representation of the relationship between the variables. His importance also lies in how he used these new tools he developed to solve some of the most important problems in mathematics and physics. In doing so Oresme made the first significant connection between geometry and algebra and their synthesis into the field of analytic geometry.

As noted above the Greeks had found they could define curves in terms of equations. But, ultimately they were limited by their lack of algebraic tools. For example, the Greek's did not have the compact equation notation we have today. This means if they wanted to describe how to produce a line they could not say $y=mx+b$, rather they would have to give a lengthy lexical description of how to construct this relationship. This led the Greeks to be fixated on the geometric rather than the analytical aspects of these equations, as a result they viewed the equations as merely by products, rather than something worth studying on their own ([3], 110).

Oresme's works on graphs, or as he refers to them "latitude of forms" is mostly documented in his *Tractatus de Configuationibus Qualitatum et Motuum (Treatise on the*

Configurations of Qualities and Motions) composed sometime in the 1350's while he was at the College of Navarre ([17], 304). From these works it is clear Oresme was motivated to develop a better understanding of the relationships between changing qualities, and to better explain a wide range of physical and psychological phenomena ([16], 1031). It is also interesting to note that there appears to be a pedagogical aspect to Oresme's work as well, as he writes:

> Something is more quickly and perfectly understood when it is explained by a visible example. Thus it seems quite difficult for certain people to understand the nature of a quality that is uniformly difform. But what is easier to understand than that the altitude of a right triangle is uniformly difform?. . . . Then one recognizes with ease in such a quality its difformity, disposition, figuration, and measure. ([12], 29)

So it is clear that Oresme's motivations are not purely mathematical in nature, but rather a reflection of his whole career identity as a theologian, teacher, and mathematician.

The *Treatise on the Configurations of Qualities and Motions* is divided into three parts, with the first and second both consisting of 40 chapters ([17], 304). It is in the first part where Oresme lays out his method, and where we will begin our analysis. Oresme begins by describing how to construct a graph. He uses the term's "longitude" and "extension" to refer to the horizontal axis of the independent variable, and the terms "latitude" and "intensity" to refer to the perpendicular vertical axis of the dependent variable ([12], 29). Note that the terms latitude and longitude Oresme uses here first come from their use in marking maps.

In order to construct a graph Oresme begins by constructing a horizontal line segment, this will represent the extension of the graph. Next, divide the segment into equal units called degrees. Next, determine the intensity of the quality at each degree, and draw a line from the extension to the intensity - this will usually be defined as some proportion of the extension. After a correspondence has been made between each of the extension points and each of the intensity points at each degree draw a line between neighboring intensity, this will give you your desired curve. Finally, connect the end points of the curve and the extension line to close the shape, and you will have a geometric figure, which Oresme refers to as the "linear configuration" of the quality ([14], 174). In this Treatise Oresme would even go so far as to suggest extrapolating this method to two variables and finding the volume of the resulting shape, though this does not appear to have been nearly as heavily studied ([4], 241).

In his work he classified three types of curves: uniform, this means constant as described in graph 3a, uniformly difform, this means change is constant such as with a line described in graph 3b, and difformly difform, or not constant and not constant in its change as described in graphs 3c ([13], 5). He makes more subtle distinctions with regards to difformly difform curves, such as distinguishing between convex and concave curves, but these are secondary characteristics. ([16], 1032). He also discovered an important fact that the curvature of a circle is inversely proportional to its radius ([16], 1032).

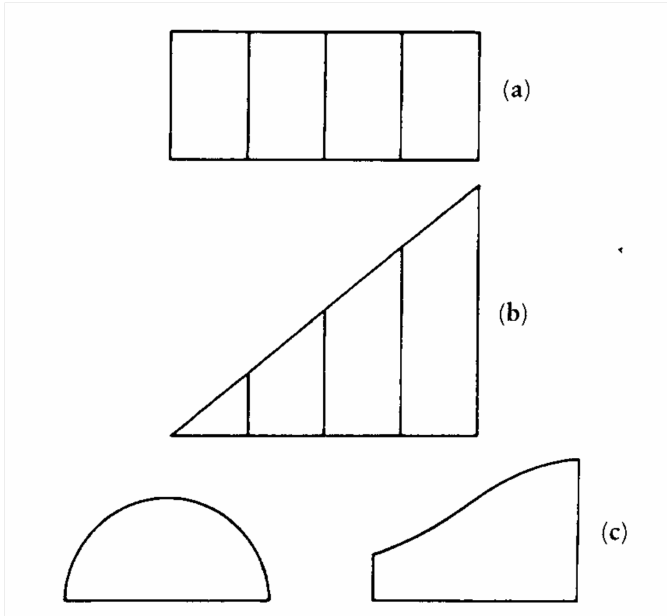

Figure 3. ([13], 5)

In section two of the *Treatise on the Configuration of Qualities* he delves into applications of this technique ([14], 174). Oresme states that everything measurable is manageable as a continuous quality, here it seems he understood the basic concept of what we today call a function, or as some historians have put it Oresme had established quasi-functional variations ([14], 110). He uses his new diagrammatic technique of graphing to solve a wide range of problems in different fields from analyzing the variable heat across a metal rod, to calculating grace in a man, to studies in music theory. It is also interesting to note that Oresme imagines a whole range of variables which could be graphed here from quantifiable measures such as position and velocity to more abstract qualities such as grace and whiteness. Oresme writes:

> One quality of the body—say, its hotness—can be figured in one way, and perhaps another quality of the same body, such as its whiteness, can be figured in another way, and perhaps another of its qualities—possibly its

sweetness—can be figured in a still different way, and similarly for the other qualities. ([12], 32)

But, in all of these studies by far his most poignant are his studies of motion.

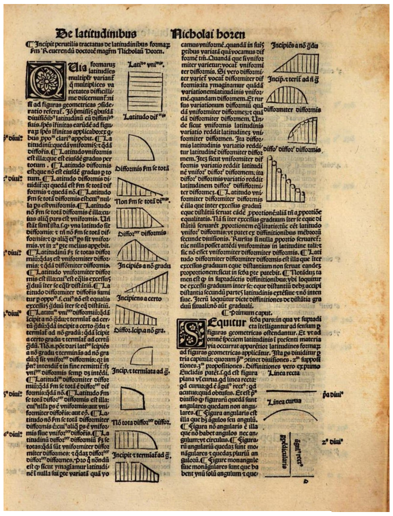
Figure 4. 1505 copy of *Tractatus de Configuationibus Qualitatum et Motuum,* [18]

Oresme used graphs in his proof of the mean velocity law, also sometimes referred to as the Merton rule. Oresme and the Oxford Calculators had been the first scientists since Archimedes to advance the study of mechanics ([3], 244). While the Merton rule had first been proved by the Oxford Calculators at Merton College in the 1330's - before the time of Oresme - Oresme's proof is significant for the history of mathematics because it is perhaps the first example in of a purely mathematical deduction of a physical phenomena ([17], 304). Oresme begins his proof by stating his proposition:

> Every quality, if it is uniformly difform, is of the same quantity as would be a quality of the same or equal subject that is uniform according to the degree of the middle point of the same subject. ([12], 30)

Or another restating, for an object traveling with constant acceleration, this distance traveled will be equal to the time covered multiplied by the velocity at the midpoint.

He proves this with the aid of figure 5. shown below - which in his terminology would be an example of a uniformly difform graph. Begin with the quality of the uniformly difform shape *ABC*. Now let the point *D* be the middle point of the extension and *E* the intensity at *D*. Now let *F* and *G* be equal to the intensity of *E* but at *A* and *B* respectively. Then by proposition 26 of Euclid's *Elements** triangles *EFC* and *EBG* are equal. Therefore triangle *ABC* is equal to rectangle *ABFG* so the area under *ABC* is equal to *AB*•*DE.* Oresme is essentially arguing that the displacement from *A* to *D* is equal to the displacement from *D* to *B* so the areas must be equal ([12], 30).

Note that though in the diagram provided the final value for the velocity equals zero, and forms a triangle, the proof is equally valid if the final velocity were not equal to zero and the resulting figure were a trapezoid.

It is interesting that Oresme thinks of this relationship as geometric in nature, and choses to describe the relationship between distance, velocity, and time in terms of the area of the resulting shape, rather than in terms of magnitude or some equation. In thinking this way he seems to anticipate what would become the fundamental theorem of calculus a few centuries later.

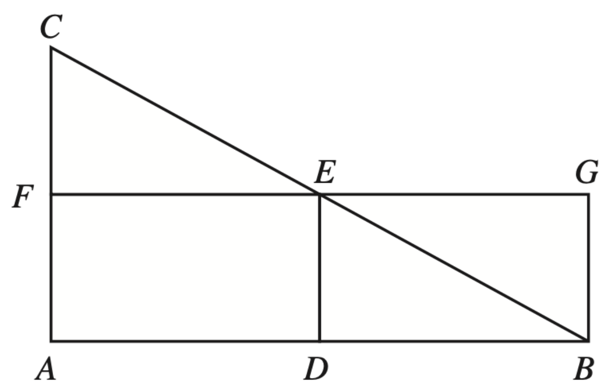

Figure 5. ([12], 30)

*Proposition 26 of Euclid's *Elements*: *If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle* ([6], 307).

Even though Western mathematics would regress in the century following Oresme's death, his work on the latitude of forms would still have a profound effect on shaping Western mathematics. This is demonstrated by the fact that Oresme's *Treatise on the Configurations of Qualities and Motions* would be printed multiple times as late as the 16th century ([4], 241). It is unclear whether or not Galileo was familiar with the work of Oresme. Master of the history of mathematics Carl Boyer argues that Galileo must have certainly been familiar with Oresme's diagrams because there are too many similarities between their works ([4], 301). Though personally this appears to be little more than a coincidence, as there are so many natural methods of constructing these types of diagrams. If it is a coincidence it would not be the first time in the history of mathematics two similar geometric constructions were discovered independently - such as with Blaise Pascal and medieval Chinese mathematicians both completely independently discovering Pascal's triangle - also according to mathematician Dan Mumford there are no records of Galileo citing Oresme ([13], 14). Though it is also possible Oresme's work may have been disseminated to Galileo indirectly. Regardless of whether or not Galileo was directly familiar with Oresme's work there appears to be a direct intellectual lineage from Oresme's proof of the Merton rule to Galileo's work on falling bodies.

After reading this section one may conclude Oresme invented analytic geometry and the study of kinematics. However, this would simply not be accurate. While his strides in these fields represent monumental breakthroughs Oresme's work still lacks many of the key insights and much of the proper mathematical rigor expressed by Fermat, Descartes and Galileo in later centuries. Though this is not necessarily the fault of Oresme, as the mathematical tools of his generation were still rudimentary, and he was limited by the scholastic traditions of his era. To quote historian Dana Durand "The genius of Oresme's mind lay in its facility for combining ideas, for detecting inter-relations between fields of thought which more pedestrian thinkers had failed to note ([14], 178)." The works of Oresme and the men at Merton were monumental leaps in Western thought and demonstrate the West developing a new sense of mathematical and scientific tradition distinct from the ancient world, which had been looming over the continent for nearly a millennium.

Conclusion

Between the fall of the Western Roman Empire to the life of Oresme the West had experienced a distinct lack of scholastic progress due to the calamitous political state of the continent. However, between the 13th and 14th centuries Europe saw an anomalous uptick in manuscripts published, and great works of artistic and intellectual development. Between these centuries Europe produced the poetry of Dante, Petrarch and Chaucer, the art of Duccio, and the mathematics of Fibonacci, Suiseth and Oresme ([16], 1031). It is also during this time much of Western Europe undertook massive scale infrastructure projects, building clock towers and cathedrals, most notably this is when France completed the Notre Dame Cathedral ([13], 1). These centuries constitute the one bright spot in medieval mathematics. But for as quickly as it seemed mathematics had been revived in the West it regressed again in just a few generations.

If the development of mathematics had followed a straight line there would have been another generation of mathematicians immediately following Oresme to expand upon his work, and this century would have been seen as the dawn of a revival of mathematics. Unfortunately, history has demonstrated progress is anything but linear. Due to the Black Death and the Hundred Years' War many scholars in Britain and France were forced to work in isolation ([16], 1033). This limited distribution of manuscripts and correspondence among universities, and as a result Oresme did not form a heritage or some school of thought. Because of the lack of a heritage following his death Oresme is often relegated to a secondary character in the history of mathematics. But this paper demonstrates that Oresme was an ancillary character in the development of Western mathematics in an age with little mathematical progress. His solutions to infinite series were as brilliant and creative as Euler, his development of graphical methods shifted Western mathematics away from the ways of the ancients, and and his work on motion was a significant precursor to Galileo's two centuries later.

delete extra "and"

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