Descriptive Title: Math 340 History of Mathematics Syllabus
Audience: Mathematics Majors
Trinity International University Spring 2013

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1:15 – 2:05 Wednesday
10:50 - 12:05, 1:40 - 2:55 Thursday
1:15 – 2:05 Friday

Text: *The History of Mathematics: An Introduction*, 7th ed. (2011), by David M. Burton (previous editions have very different page numbers, so don't buy them)

Course description: MA 340 History of Mathematics
Includes Greek mathematics, non-western mathematics, the development of calculus, mathematics of the 18th and 19th centuries, non-Euclidean geometry, and set theory. Prerequisite: MA 121 or consent of instructor. Offered spring semester in odd-numbered years. Four hours.

Trinity's mission: Trinity International University educates men and women for faithful participation in God's redemptive work in the world by cultivating academic excellence, Christian faithfulness, and lifelong learning.

The mission of the Department of Mathematics and Computer Information Systems is to
- relate mathematics to the world the Lord has made and to the technology developed to deal with this world
- train students to think logically and analytically about mathematical or computer-related questions
- give students the tools they need to solve mathematical or computer-related problems
- treat students with dignity and model godly living
- advise students as they prepare for service in mathematical fields
- demonstrate how mathematics gives us insight into the Christian faith and vice versa .

Course objectives: Upon the completion of this course, you should be able to
- explain the mathematical achievements of the Egyptians, Babylonians, Greeks, Chinese, Arabs, and (Asian) Indians
- compare the astronomical beliefs of Ptolemy, Copernicus, Kepler, and Newton
- trace the development of calculus
- list some of the achievements of Newton, Euler and Gauss
- explain the distinctives of non-Euclidean geometry
- state the definition of the limit
- explain the achievements of Cantor and others in the field of set theory
- explain the significance of Gödel's incompleteness theorems
- describe certain mathematical achievements of the past century

Our class meets 10:00 - 10:50 MWF in Aldeen 202 and 12:15 – 1:30 Tu in McLennan 213.
Your grade will be based on the following:

3 Exams (100 points apiece)	300 points
≈40 Quizzes (6 points apiece)	≈240
Essay	100
Oral Presentation	25
Homework	100
Films	60
Final	120

Total ≈945 points

Our exams will be on Wednesday, February 6, 2013; Wednesday, February 27, 2013; and Friday, April 12, 2013. Notify me beforehand if you will be absent on the day of a test. We will have a quiz every day, except on days when there was no reading assignment due. The quiz will cover the reading assignment for that day. On the quiz, you will also be asked what percentage of the assigned reading you did for that day. I can give you a make-up quiz if you are absent, but you will have to make it up within a week. Apart from a few homework assignments given in class, all of the homework assignments for this course are listed at the end of this syllabus. Homework needs to be turned in promptly (the next time we meet).

Sometime this semester, you will need to watch three of these four films (all are available on DVD and can be checked out from Rolfing Library):

· *Infinite Secrets: The Genius of Archimedes* ("In 1991, a small Medieval prayer book was sold at auction. Miraculously, some original writings of Archimedes, the brilliant Greek mathematician, were discovered hidden beneath the religious text. Through scholarly detective work with the help of modern technology, this book now reveals Archimedes' stunningly original concepts, ideas, and theories—revelations that, if known sooner, might have reshaped our world." http://www.amazon.com/NOVA-Infinite-Secrets-Genius-Archimedes/dp/B0001WTX7S)

· *A Brilliant Madness* ("This is the story of a mathematical genius whose career was cut short by a descent into paranoid schizophrenia At the age of 30, John Nash, a stunningly original and famously eccentric MIT mathematician, suddenly began claiming that aliens were communicating with him and that he was a special messenger. Diagnosed with paranoid schizophrenia, Nash spent the next three decades in and out of mental hospitals, all but forgotten. During that time, a proof he had written at the age of 20 became a foundation of modern economic theory. In 1994, as Nash began to show signs of emerging from his delusions, he was awarded a Nobel Prize in Economics." http://topdocumentaryfilms.com/a-brilliant-madness-john-nash/)

· *Galileo's Battle for the Heavens* ("This is the story of the father of modern science and his struggle to get Church authorities to accept the truth of his astonishing discoveries. The program is based on Dava Sobel's bestselling book, Galileo's Daughter, which reveals a new side to the famously stubborn scientist—that his closest confidante was his illegitimate daughter, Sister Maria Celeste, a cloistered nun.") http://www.pbs.org/wgbh/nova/ancient/galileo-battle-for-the-heavens.html

· *Ramanujan: Letters from an Indian Clerk* (This is "the extraordinary story of how in 1914 the self-taught [mathematical] genius SRINIVASA RAMANUJAN was brought from Madras to Trinity College, Cambridge, by the great English pure mathematician GH Hardy, who called their relationship 'the one truly romantic episode of my life' ") This is not available at Rolfing, but it is on YouTube:
https://www.youtube.com/watch?v=OARGZ1xXCxs

Our final will be on Wednesday May 8, 2013 from 10:30 a.m. to 12:30 p.m. The final will not be cumulative, apart from some things I will tell you about beforehand.

Academic Dishonesty: Trinity considers academic dishonesty in the forms of cheating and plagiarism to be serious academic infractions and a breaking of college Community Expectations. Cases of deliberate cheating or plagiarism will be reported to the Academic Dean; you will receive a failing grade for the assignment and may even receive a failing grade for the course. In such a case, you may not drop the course. For additional information see Academic Integrity in the Official Policies section of this catalog, and consult the Student Handbook.

When we study Greek geometry in about two weeks, you will need a compass and a ruler.

Free math tutoring is available in the University Student Success Center. No appointment is necessary; just walk in.

Sources on reserve in the library:
A History of Mathematics, Carl Boyer, QA 21.B767x1991
A History of Mathematics: An Introduction , Victor Katz QA 21.K33 1998
The Norton History of the Mathematical Sciences: the Rainbow of Mathematics, I. Grattan-Guinness, QA 21.G695 1998
The History of Mathematics: A Brief Course, Roger Cooke, QA 21 .C649 1997

Math 340 History of Mathematics
Essay
Due Monday, April 8, 2013

Write a 7- to 9-page essay on some topic in the history of mathematics (7-9 pages does not include a mostly blank title page or the bibliography). You may research a mathematical idea and write a paper tracing its development over time. Or you may write about the contributions made by a significant mathematician (in this case, you should focus on this person's achievements rather than present a mere biography of the mathematician).

Possible topics include the mathematics of Egypt or Babylonia; the Pythagoreans; Euclid's Elements; Archimedes; Diophantine equations; the mathematics of India, China or the Arab world; the introduction and acceptance of negative numbers or Arabic numerals; Fibonacci numbers; Kepler; Newton's Principia; the dispute between Newton and Leibniz over the invention of calculus; Fermat's Last Theorem; the Goldbach conjecture; Euler; Gauss; the development of non-Euclidean geometries; Hilbert; Gödel; prime numbers; π; e; the Golden Ratio φ, Cantor; the contributions of female mathematicians like Kovalevsky, Noether, Julia Robinson; etc. This list of topics is not meant to be exhaustive.

Your sources may be books on the history of mathematics; biographies of mathematicians; articles in the American Mathematical Monthly, Scientific American, Science (journals available in our library) or similar journals; etc. Our library has many mathematics books; they usually have call numbers beginning with QA. If you want more ideas for essays, you can browse in this section of the library. Several sources should be a printed sources (besides a standard encyclopedia or our textbook), at least one source should be an Internet source, and at least one source should be a journal article. Do not use Wikipedia or a similar online encyclopedia as a source. Do not rely too heavily on our textbook.

Here are a few helpful hints about writing essays:

- Give yourself plenty of time to think about what you want to write and to write the essay. Do not try to throw something together at the last minute.
- Do not rely too heavily on just one or two sources.
- If you use something from the Internet, make sure it comes from a reliable source.
- Before you begin to write, it may be helpful to think about what you want to say and to make an outline. This will help you to organize your thoughts.
- Proofread your paper before turning it in. You may even want to read it aloud.
- You must have someone else proofread your paper, someone who is a good writer and will give you honest feedback. You can get such assistance at the University Student Success Center; it is good to make an appointment in advance.
- Use the Math Research Guide (it is available online at the Rolfing Library home page under Research Guides). Through it, you can access journal articles, which are available through JSTOR (these are all full text), Wilson Select Plus (these are all full text), Academic Search Premier, and Google Scholar. Google Books is a helpful resource, too. If you want a particular article but cannot obtain it in this way, you might be able to get it through Interlibrary Loan; that can take a couple of weeks, so start early.

Your essay should be double-spaced and have standard margins and font (12pt) and spacing. If you write exponents, use exponential notation (e.g., x^2) instead of caret notation (e.g., x^2). If you want to show subtraction, use a minus sign (−) instead of a hyphen (-); the minus sign is available under Insert, then Symbol. Or use Insert, then Equation to write your equation.

One purpose of this exercise is to help you learn to write correctly. You will not receive a grade on your paper until it is intelligible and completely free of errors in spelling, punctuation and grammar.

If you are an education major, you may want to place this paper in your professional portfolio.

You will also make a 6-to-10-minute oral presentation to the class based on your paper. The first presentation will take place a few days after you turn in the essay, and we will have one or two presentations every class period. The essay is worth 100 points and the oral presentation is worth 25 points.

MA 340 History of Math Assignments Spring 2013

from *The History of Mathematics: An Introduction*, 7th ed. (2011), by David M. Burton

Due on	Assignment #	Due on	Assignment #
Wednesday, January 16, 2013	1	Monday, March 11, 2013	
Friday, January 18, 2013	2	Wednesday, March 13, 2013	17
Wednesday, January 23, 2013	3	Friday, March 15, 2013	18
Friday, January 25, 2013	4	Monday, March 18, 2013	19
Monday, January 28, 2013	5	Friday, March 22, 2013	20
Wednesday, January 30, 2013	6	Monday, March 25, 2013	21
Friday, February 1, 2013	7	Wednesday, March 27, 2013	22
Monday, February 4, 2013	8		
Exam 1:		Essay due:	
Wednesday, February 6, 2013		Monday, April 8, 2013	
Friday, February 8, 2013	9	Wednesday, April 10, 2013	23
Monday, February 11, 2013	10		
Wednesday, February 13, 2013	11	Exam 3:	
Friday, February 15, 2013	12	Friday, April 12, 2013	
Monday, February 18, 2013	13		
Wednesday, February 20, 2013	14	Monday, April 15, 2013	24
Friday, February 22, 2013	15	Wednesday, April 17, 2013	25
Monday, February 25, 2013	16	Friday, April 19, 2013	26
		Monday, April 22, 2013	27
Exam 2:		Wednesday, April 24, 2013	28
Wednesday, February 27, 2013		Friday, April 26, 2013	29
		Monday, April 29, 2013	30
		Wednesday, May 1, 2013	31
		Friday, May 3, 2013	32

The only homework you will turn in is prefaced by "Do," "Find," "Draw," "Write," etc. So, for example, from Assignment 1 you will turn in Secn 1.2 1adf, 2abd, 3ab, 11acdf, 12aef but you will not turn in the answers to "Look for tally sticks, quipu, Herodotus, ...What are the differences between the hieroglyphic number system and the hieratic number system?" But you will want to make a note of the answers to these questions, because our daily reading quizzes will be based on them. Our reading quizzes will <u>not</u> be based on the homework problems you do; you will be tested on your understanding of homework questions in our exams, not in our reading quizzes. The material in parentheses is what we will cover in class.

1. (Egyptian numerals)
Read pp. 1-16 (up to "... positional principle.").
Look for tally sticks, quipu, Herodotus, <u>Histories</u> of Herodotus, hieroglyphics, papyrus, hieratic script, demotic script.
What did the Egyptians use their mathematics for?
Why didn't peoples who lived before the Egyptians develop sophisticated number systems?
What are the differences between the hieroglyphic number system and the hieratic number system?
Do this homework from Secn 1.2: 1adf, 2abd, 3ab, 11acdf, 12aef.

2. (Egyptian algorithms for multiplication and division, Egyptian fraction notation, Egyptian "algebra")

Read Secn 2.1.
Read Secn 2.4 up to p. 55 ("... by the Egyptians").
Read pp. 57-58 ("Speculations About ... of 1° ").
Read the last paragraph of Secn 2.4.
Look for Déscription de l'Egypte, Rhind papyrus, Rosetta stone.
Note the inaccuracy of Egyptian geometry.
Do this homework from Secn 2.3: 1ab, 2be (express your answers using unit fractions), 9, 20ab (use false position to solve).
Do this homework from Secn 2.4: 4.

3. (Babylonian numerals, Babylonian square root algorithm, form groups for take-home quiz)
Read Secn 1.3.
Read Secn 2.5 up to and including the table on p. 67.
Look for cuneiform, sexagesimal.
Why do we know more about Babylonian mathematical achievements than about Chinese mathematical achievements of the same time period?
Note the differences between the ways the Egyptians, Babylonians and the Chinese recorded their information.
How was Babylonian mathematics more sophisticated than that of the Egyptians? List at least two ways. What are other differences between Egyptian and Babylonian mathematics?
Do this homework from Secn 1.3: 1abd, 2ab, 4ab(the answer to 4b in the back of the book is wrong: it should be 12 1/16 instead of 12 1/6), 11ab, 10ab.
Do this homework from Secn 2.3: 6, 20c (use false position to solve).
Do this homework from Secn 2.4: 2.

Using several iterations of the Babylonian algorithm for calculating square roots, estimate $\sqrt{7}$.

Group-take home quiz due the class after next: Secn 1.3: 13ade, 14ab.

4. (Thales, measurement of the Great Pyramid, Greek alphabet)
Read Secn 3.1.
Note Thales and his accomplishments.
Note how Greek math differed from Egyptian math (list three ways).
Note where the Greeks lived.
Do this homework from Secn 1.3: 1f.
Do this homework from Secn 2.3: 7.
Do this homework from Secn 2.6: 1, 2.
Learn the first 8 letters of the Greek alphabet on p. 761 (name, upper-case and lower-case forms)

5. (Triangular and square numbers, formula for the nth pentagonal number p_n)
Read Secn 3.2 up to the second diagram on p. 97.
Look for Pythagoras, Pythagoreans, "Everything is number," quadrivium, trivium, triangular numbers, square numbers.
Note the Pythagoreans' ideas, superstitions and educational practices.
Learn the next 8 letters of the Greek alphabet on p. 761 (name, upper-case and lower-case forms).
Do the following exercise:

> When held perpendicular to the ground, a stick which is 4 feet 10 inches long casts a shadow which is 11 feet 5 inches long. At the same time, a TV antenna casts a shadow which is 30 feet 5 inches long. Determine the height of the antenna. Give an answer accurate to the nearest inch.

Find a formula for the nth hexagonal number h_n.

6. (Irrational numbers, $\sqrt{2}$ is irrational, Eudoxus, Plato)
Read from p. 101 ("Zeno's Paradox...") to end of Secn 3.2.
Read pp.109-110 ("The Crisis... assumption").

Read from p.116 (“Eudoxus... “) to end of Secn 3.3.
Look for Zeno’s paradoxes (especially the race between Achilles and the tortoise).
Note how the Greeks responded to the discovery of irrational numbers.
Do this homework from Secn 3.2: 1, 6.

Prove that $\sqrt{3}$ is irrational.

Using methods similar to those of Thales, find the height of the flagpole in front of the Mansion. Be careful: some of the ground near the flagpole is not level.
Learn the last 8 letters of the Greek alphabet on p. 761 (name, upper-case and lower-case forms).

7. (Pythagorean Theorem and its proof, Pythagorean triples, infinitude of primes, Greek numerals, gematria, Greek fractions, Zeno’s paradoxes)
Read Secn 3.4 up to p. 122 (“...of thought.”).
Read p.124-125 (“The Duplication...Delian problem.”).
Read p.126 (“The Trisection... carried out”).
Read first 3 paragraphs of Secn 3.5.
Read from p.134 (“The Grove...”) to the end of Secn 3.5.
Look for the three famous construction problems of antiquity, sophists, Plato’s Academy (and the inscription above its door), the Museum.
Do this homework from Secn 3.2: 3, 8.
Do this homework from Secn 3.3: 4.

Group-take home quiz due the class after next:

a. Prove that $\sqrt{11}$ is irrational.

b. Convert your name to Greek letters, calculate the corresponding values of the Greek letters, and add up the values.
c. Convert your name to Hebrew letters, calculate the corresponding values of the Hebrew letters, and add up the values.

8. (Parallel Postulate)
Read Secn 4.1 and 4.2 up to p. 148 (“... and no undefined terms”).
Read Secn 4.3 up to p. 173 (“... fact follows”).
Look for Euclid, Elements, Alexandria, the Museum, the Museum library.
Note the format of Euclid’s Elements.
Do this homework from Secn 4.3: 1a, 21ab (in 21b find just two more primes).
Do this homework from Secn 3.3: 2.

9. (Ptolemy’s epicycles, Eratosthenes’ measurement of the earth, the five convex regular polyhedra, Euclidean algorithm)
Read Secn 4.4.
Look for Eratosthenes, “Beta,” Sieve of Eratosthenes, Ptolemy, Almagest, epicycles.
Note how Eratosthenes measured the circumference of the earth.
Note how Ptolemy’s treatise on geography influenced Columbus.
Do this homework from Secn 3.3: 15.
Do this homework from Secn 4.3: 15ab.
Draw a line segment. Using just a compass and a straightedge, figure out a way to construct the perpendicular bisector of the segment.

10. (Method of exhaustion, integrative question about theological and mathematical knowledge, integrative question about the deductive method of apologetics)
Read Secn 4.5, but skip p. 198-199 (“The problem can ... is known.”), skip the proof of Proposition 2, and skip pp. 202-204 (“To give... 1647").

Look for “Eureka!”, method of exhaustion, Archimedes’ inventions and devices.
Draw a line segment, and pick a point on the segment. Using just a compass and a straightedge, figure out a way to construct a segment perpendicular to the original segment through that point.
Do this homework from Secn 4.3: 5.
Group-take home quiz due the class after next: Secn.3.3: 17.

11. (Integrative question on the value of pi, Archimedes palimpsest and handout, completing the square)
Read Secn 5.1, 5.2 up to p. 221 (“... 16 − 256/25 = 144/25.")
Read Secn 5.4 up to p. 236 (“...Latin-speaking West.”).
Read the handout I’ll give you on Archimedes’ Method.
Look for Mathematical Collection of Pappus, Diophantus’s Arithmetica,
Note why the Greeks accomplished less as time passed.
Note why the Romans did not make any great mathematical achievements.
Note the fates of Hypatia and the Museum library at Alexandria.
Do this homework from Secn 5.3: 1, 4.
Look at the ancient coins in the display case outside the computer lab in Rolfing Library.

12. (Christianity vs. paganism, Hypatia’s fate, the mathematics of the Arabic world and India--Aryabhata, Brahmagupta, Bhaskara, abacus)
Read Secn 5.5 but skip p. 241 (“For.... −p/2"), skip pp. 242-243 (“As would be... y = 5 + 3 = 8"), and skip pp. 244-249 (“Another prominent... an Arabic mathematician”).
Look for caliph, Damascus, Baghdad, al-Khowarizmi, the origins of the words “algebra” and “algorithm,” Liu Hui, Nine Chapters on the Mathematical Art.
Note how Arab math was different from Greek math (list two ways).
Note how Chinese math was different from Greek math (list two ways).
Note how the Chinese accepted and represented negative numbers.
Note why Chinese mathematics suffered a decline.
Do this homework from Secn 5.5: 1ab. Draw pictures to accompany your solutions.
(This assignment is long.)

13. (Mathematics in the Middle Ages, Proclus, Boethius, Fibonacci, Liber Abaci , Fibonacci numbers, constructible numbers)
Read Secns. 6.1 and 6.2 up to p. 283 (“... by the Arabs”).
Look for Charlemagne, Alcuin, Leonardo of Pisa (Fibonacci), Liber Abaci.
Note the revival of education.
Note through whom Greek classics were reintroduced into Western civilization.
Note the development of “Arabic” numerals and their introduction into the West.
Do this homework from Secn 5.5: 1c. Draw a picture to accompany your solution.

14. (Trigonometry, Regiomontanus, Pacioli’s Summa, Perspective)
Read Secn 7.1.
Note the disasters in Europe during the 1300s and 1400s, especially the plague.
Note why there was a renewed interest in Greek classics (give two reasons).
Note the development of universities and of printing.
Note the recovery of ancient manuscripts.
Look for Regiomontanus, Pacioli.
Look at each of these four web sites. Some may be useful to you as you write your paper. Print out and turn in (or email me) something from each web site to show that you visited it:

- MacTutor History of Mathematics Archive (note that the address is www-groups and not www.groups)
http://www-groups.dcs.st-and.ac.uk/~history/
- Convergence (online journal about the history of mathematics with pictures of mathematicians and of famous books)
http://mathdl.maa.org/mathDL/46/
- British Society for the History of Mathematics links to web sites on the history of mathematics
http://www.dcs.warwick.ac.uk/bshm/resources.html
- Math Forum (choose Resources and Tools, then Internet Math Library, then History and Biography)
http://mathforum.org/

15. (Solution of the cubic--then and now, Bombelli)
Read Secns 7.2 and 7.3 up to p. 324 (".. a scoundrel"), but skim the equations on pp. 321-324.
Read Secn 7.4 up to p. 329 ("... stronghold").
Read from p. 331 to the end of Secn 7.4 ("Our story... ").
Note advances in notation (+, −, Vieta's use of letters).
Note how and why mathematicians jealously guarded their results.
Summarize Tartaglia's dispute with Fiore, Cardan and Ferrari.
Do these problems on the Fibonacci sequence from Secn 6.3: 1, 2a, 3, 5.
What did Ruffini and Abel show?
Why was Abel's proof initially rejected?
How old was Galois when he first submitted his proof?
What circumstances prevented Galois' proof from being published?
How old was Galois when he died?
How did he die?
Do this homework from Secn 7.3: 3a.
(This assignment is long.)

16. (Napier's logarithms, our logarithms, logarithm tables, slide rules, reliability of Web sites)
Read Secn 8.1 up to p. 353 ("... sine could attain.")
Read pp. 354-355 ("Napier's *Descriptio* ... virtually unnoticed.")
Look for John Napier, Henry Briggs.
Note how Galileo contradicted prevailing astronomical theories (list two ways).
Note how the Catholic Church treated Galileo.
Note the development of notation (=, <, >, √, exponents, decimal fractions).
Do this homework from Secn 7.3: 3b, 5.
Do these problems on the Fibonacci sequence from Secn 6.3: 6, 7a (hint for 7a: use Problem 6, even if you could not do it).
Look at these two web sites:
http://mathcentral.uregina.ca/QQ/database/QQ.09.01/andy1.html
http://www.uwlax.edu/faculty/hasenbank/mth320/notes/Chapter%209%20-%20The_Extraordinary_Sums_of_Leonard_Euler.pdf
Tell me what makes you suspect that these web sites not as reliable as the web sites you looked at in the last assignment.
Use the logarithm table I will give you to approximate 497 × 639

17. (Kepler's three Laws of Planetary Motion, Galileo's influence on science)
Read pp. 355-369 ("The Astronomical... 1692").
Note how exponents and a forerunner of the coordinate system were found in Descartes's Discourse.
Note Descartes' famous quote and what it means.
Note how Descartes died.
Look for Johann Kepler, Tycho Brahe, Kepler's three Laws of Planetary Motion.
Do these problems on the Fibonacci sequence from Secn 6.3: 7bd (Hint: Use Problem 6), 8.
Use the logarithm table I gave you to approximate 3450 × 986.
Determine the average distance from the sun to

(a) Pluto, which takes 247.7 years to orbit the sun
(b) Mercury, which takes 88.0 days to orbit the sun.

Given that the average distance from Neptune to the sun is 2 794 000 000 miles, find its period.
Drop by my office sometime over the next week for a quiz on your use of the abacus and the slide rule.
Look at this fun Web site:
http://mathstamps.org/links.html

18. (Biography of Newton)
Read Secn 8.3 up to p. 384 ("...lost to it."), pp. 386-389 ("The Lucasian Professorship... in the preparation"), pp. 391-393 ("All through... by himself").
Read from p. 395 ("The well-known...") to the end of Secn 8.3.
Look for Isaac Barrow, Robert Hooke, Lucasian chair.
Note which of William Oughtred's and Thomas Harriot's notation survives to the present day; also note when it was first used.
Note Newton's three major accomplishments during his two-year hiatus from college.
Homework to turn in: Construct a timeline of Newton's life (it does not have to be long).
(This assignment is long.)

19. (Calculus)
Read Secn 8.4, but skip pp. 413-420 ("Leibniz's Creation... severe criticism").
Look for Marquis de l'Hospital, Maria Agnesi.
Write a summary the dispute involving Newton, Leibniz and their followers, and note its impact on English mathematics (this is homework you will turn in).
Find the slope of the line tangent to
(a) $y = x^2$ at (4, 16)
(b) $y = 5x^3$ at (2, 40)
(c) $y = x^4 + 7x + 8$ at (2, 38).
Find the area of the region between the x-axis and
(a) $y = x^2$ for $0 \leq x \leq 5$
(b) $y = 5x^3$ for $1 \leq x \leq 3$
(c) $y = x^4 + 7x + 8$ for $-1 \leq x \leq 2$.

20. (Probability, Pascal's Triangle, Plagiarism)
Read Secn 9.1 up to p. 452 ("... was permanent").
Note what two very different subjects gave birth to the field of probability.
Note the faulty mathematics behind the annuities of several centuries ago.
Note how long dice, games of chance, and playing cards have been around.
Note how and why Pascal's father kept him from learning mathematics.
Look for mortality table, Blaise Pascal, Father Marin Mersenne's "academy," Charles Babbage, Babbage's Difference Engine, Pascal's conversion, Pascal's Pensees .
Using Pascal's Triangle,
(a) expand $(x + y)^7$
(b) expand $(x + y)^8$
Using the formulas we learned in class,
(c) find the coefficient of $x^{80}y^3$ in $(x + y)^{83}$
(d) find the coefficient of x^2y^{100} in $(x + y)^{102}$
(e) find the coefficient of x^{36} in $(x + 2)^{40}$
Please do this plagiarism tutorial, including the pre-test, quizzes and post-test.
http://www.lib.usm.edu/legacy/plag/plagiarismtutorial.php#ti
This tutorial will be regarded as a quiz, so please do it.

21. (Integrative question on Pascal's wager, finding journal articles from the library's online databases, Mathematical Induction, Game Theory, Prisoner's Dilemma, John Nash)
Read Secn 9.3 up to p. 485 ("... = 1 / 78").
Note James Bernoulli, John Bernoulli and their accomplishments.
Note l'Hospital's financial arrangement with John Bernoulli.
Note who discovered "l'Hospital's Rule."
Look for Mécanique Céleste.
Note Laplace's relationship with Napoleon.
Do this homework from Secn 9.2: 1 bd.
Other homework to turn in:

2. If we roll a pair of dice 3 times, what's the probability of rolling at least one 7?
3. How many times do we have to roll a pair of dice before we have a better than even chance of rolling
   a. at least one 7?
   b. at least one 8?
4. Using the formulas we learned in class, find the coefficient of x^{48} in $(2x + 3)^{51}$.

22. (Perfect numbers)
Read Secn 10.1, but skim the proof of the lemma on p. 505 ("An Example...make *n* a perfect number.")
Note Father Marin Mersenne, Mersenne's academy, Mersenne primes, perfect numbers, amicable pairs of numbers.
Note how the oldest scientific societies like the Royal Society of London get their start.
Note how Frank Cole got a standing ovation.
Note the discovery of the 16-year old Italian schoolboy.
Do this homework from Secn 9.2: 1f.
Show that 28 and 496 are perfect numbers by listing and summing their divisors.

23. (Fermat's Last Theorem)
Read Secn 10.2 up to p. 514 ("... famous last theorem").
Read pp. 516-517 ("The Famous Last Theorem..... in nonzero integers").
Read p. 519 ("We might... had been doing").
Look for Fermat's Last Theorem, Wolfskehl Prize, Andrew Wiles.
Note Fermat's profession, his reluctance to publish, the fields of mathematics he cofounded.
Know the story about his note in a margin.
Do this homework from Secn 9.2: 4.

24. (Seven Bridges of Königsberg Problem, graph theory, Eulerian paths and circuits, Fumbler Rules of Grammar, Wolfskehl and Euler Path handouts)
Read pp. 520-531 ("Because of the heritage... limits and convergence").
Read pp. 534 ("In 1736... exactly once").
Read from p. 537 to the end of Secn 10.2 ("Many problems...").
Read the handout I will give you about Wolfskehl.
Look for Enlightenment, salons, Goldbach Conjecture, Königsberg Bridge Problem
In the 1700s, how did scientists earn their living?
Note George Berkeley's complaint against calculus.
Note Euler's mathematical accomplishments (list 3).
Note Euler's handicap.
Do the problems on the Euler Path handout that I will give you in class.
Homework to turn in: Construct a timeline of Euler's life (it does not have to be long).

25. (Least squares estimate, Berkeley's complaint and a related integrative question, Newton's Method)
Read Secn 10.3 up to p. 551 ("... doctoral dissertation").
Know the story of how Gauss quickly added 1+2 + ... + 100.
Look for Joseph Louis Lagrange

Make a list of Gauss's accomplishments (include those involving his dad's payroll, the 17-sided polygon, the asteroid). This is not homework to turn in.

Solve these problems using Newton's Method:

a. Find a root of $x^3 - 4x - 4 = 0$ near $x = 2$.
b. Find a root of $2x^3 - 6x^2 + 4x + 1 = 0$ in the interval $[-1, 0]$.
c. Find both roots of $x^4 - 5x^3 - x^2 + 4x + 3 = 0$ accurate to 9 decimal places.
d. Find $\sqrt[3]{40}$ accurate to 9 decimal places.

26. (Saccheri quadrilaterals, non-Euclidean geometry, taxicab geometry, indirect proof)

Read Secn 11.1 up to p. 569 ("... pure genius"), but skip the Theorem and skim the formulas.
Read pp. 598-600 ("Models of the New...1 and 2 of Euclid").
Read pp. 602-607 ("The non-Euclidean geometries... articles by Cauchy").
Look for Playfair's Axiom, Saccheri, Cauchy, Bernhard Riemann, Felix Klein, Grace Chisholm Young.
Note Cauchy's political problems.

27. (Definition of limit, counterexamples from analysis, Klein bottle, Möbius strip)

Read pp. 614-620 ("The Father of ... quasi-geometric proofs").
Read from p. 621 ("The interest... ") to the end of Secn 11.3.
Look for Karl Weierstrass, Weierstrass's definition of limit, Sonya Kovalevsky, David Hilbert, Foundations of Geometry, consistency, Kurt Gödel, Gödel's Incompleteness Theorems.
How old was Weierstrass when he became a professor?
Note the obstacles Sonya Kovalevsky faced because she was a woman (list at least 3).
Note the subject of Hilbert's talk at the 1900 mathematical congress in Paris.
Homework to turn in: We know that the limit of x^3 as x approaches 2 is 8. Find the largest value of δ (where δ is a power of 10) which works if

a. $\varepsilon = .01$
b. $\varepsilon = .001$.

28. (Complex numbers, Gödel's Incompleteness Theorems)

Read Secn 11.4 up to p. 643 ("...indicate matrices.").
Look for Tripos, Senior Wrangler, Charles Babbage, the development of complex numbers, Arthur Cayley, quaternions, Josiah Willard Gibbs, the development of matrices.
Note how Gauss modified the approach of Wessel and Argand.
Note where Hamilton was when he got his insight.
Note how Gibbs modified Hamilton's quaternions, thus inventing vectors.
Homework to turn in: Calculate the following by hand and show your work.

a. $(1 + 3i)(2 + 4i)$
b. $(2 + 5i)^2$

29. (Cantor and his set theory, integrative question on Cantor and infinity, countability of **Q**, uncountability of **R**, integrative question related to the paradox of the number of irrationals)

Read Secn 12.1.
Read Secn 12.2 up to p. 684 ("... is countable").
Look for:

Sylvester (the obstacles he faced, his influence on American mathematics),
Cantor (his revolutionary ideas, their reception, the effect on him),
Kronecker (his personality, his attitude toward Cantor).

What prompted Germany to strengthen its educational system?
How was the U. of Berlin different from other universities?

Note how Americans went overseas to learn how to do mathematical research.
Note how Johns Hopkins and the U. of Chicago were the first American universities to emphasize mathematical research.

30. (Russell's Paradox, symbolic logic, algebraic and transcendental numbers)
Read pp. 689-690 ("Transcendental Numbers... transcendental, not algebraic").
Read pp. 692-694 ("Theorem... current mathematical techniques").
Read Secn 12.3 up to p. 699 ("... prolific of mothers").
Look for algebraic numbers, transcendental numbers.
Note who proved that e is irrational and when this was proved.
Note when π and e were proved to be transcendental.
Why is it significant that π is transcendental?
Homework to turn in: Make a chart showing when we calculated π to 1000 digits, 2000 digits, etc. Using the Web, determine how many digits of π are known today.

31. Read Secn 13.1, but skim p. 723 ("Hardy's research... finite value").
Look for G.H Hardy, Riemann zeta function, Riemann Hypothesis, Waring's Problem, Ramanujan, partition function $p(n)$, Ramanujan's notebooks, Ramanujan's "lost notebook."
What effect did the Tripos have on English mathematics and why?
What country was Ramanujan from?
How much of college education did Ramanujan have?
How old was Ramanujan when he died?
Know the story about the taxi numbered 1729.
Note that Ramanujan had a series formula for π.

32. (Ramanujan, Mandelbrot Set)
Read the last two paragraphs on p. 734.
Read Secn 13.3 except pp. 738-739 ("Historically... mathematics graduate student").
Look for the Four-Color Theorem, the two men who proved it, how they proved it, and how long the problem went unsolved.
Look for the Kepler Conjecture and Tarski's circle problem.
Note the obstacles women faced and the obstacles women no longer faced in Emmy Noether's time.
Note the Nazi policies which affected the universities and the impact these policies had on German universities (list at least 2 policies).
Homework to hand in: Determine if the following are in the Mandelbrot set:

a. .5
b. $.3 + .5i$
c. $-.1+.1i$.